LESSON PLAN TEMPLATE

Marie Aloia

MODULE TOPIC: Using mathematical models for engineering

BRIEF DESCRIPTION: Students will create mathematical modeling tools to solve problems.
Lesson 1: Students will use a model to optimize energy transfer for maximum efficiency.
Lesson 2: .Students will create a cost estimate and present results in a report.

OBJECTIVE(S): Students will be able to:
Lesson 1: (1) Measure the efficiency of a ramp by combining and comparing the equations for potential and kinetic energy using small model cars.
(2) Predict the velocity of the cars at various efficiencies.
Lesson 2: (1) Students will be able to create a cost estimate for a camping trip working with a collection of requirements and constraints including a budget, while optimizing cost and value.

STANDARD(S) & INDICATOR(S):
CCSS.ELA-Literacy.RST.11-4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics*.
NGSS MS-ETS1-4 (Lesson 1): Develop a model to generate data for iterative testing and modification of a proposed object, tool, or process such that an optimal design can be achieved.
NGSS 5-ETS1-3 (Lesson 2): Define a simple design problem reflecting a need or a want that includes specified criteria for success and constraints on materials, time, or cost.

MATERIALS: Notes and Assignment Instructions, ramps, matchbox cars, meter sticks and stop watches for lesson 1. Lesson 2 may include an excel spreadsheet.

BACKGROUND INFORMATION:
For lesson 1 the students are told that they will be designing a procedure to measure efficiency of a ramp. Efficiency is defined as the ratio of energy output to energy input, usually expressed as a percent, and never greater than 1, or 100%. This energy can also be modeled as the ratio of the kinetic energy produced by a machine to the potential energy in the machine. For a ramp, the height is the potential (input) and the kinetic energy, (output) is measured from the speed of the car at the bottom. The maximum possible velocity occurs when the ramp is 100% efficient, so the measured velocity will always be less. At various efficiencies the car will have different velocities. Some energy will be lost to friction as well, and may even vary with the angle of the ramp. The students will discover this. Varying the angle of the ramp they will try to find the angle with the most efficient energy transfer for this ramp.

For lesson two the students will have a “challenge” to see which team can get the most within their budget, and makes best use of bargains.

CLASSROOM ACTIVITY DESCRIPTION (LABORATORY/EXERCISES/PROBLEMS) including detailed procedures:
Lesson 1) Students are told that they will be designing a procedure that can be used to measure the efficiency of a machine. Given the equation for efficiency they will propose ways of determining and measuring the input and output energy. They will then translate this to a ramp, given potential energy equation for ramp height and the kinetic energy equation for a car rolling down the ramp. They will determine the efficiency of the ramp, and then derive an equation to predict the car’s velocity based on the possible efficiencies of the ramp.

For a ramp the maximum potential energy is:

$PE = mgh$

m= mass of the car in kg
h = height of the ramp in meters
$g = 9.8 m/s^2$

This is the theoretical maximum amount of energy, or the input energy to the ramp.
The output energy is determined by measuring the velocity of the car at the bottom of the ramp. If the ramp is 1 meter long the floor can be marked in meters and the car timed with a stopwatch. The students should do three to five trials and average them for the value for velocity. The actual kinetic energy is calculated as:

$KE = \frac{1}{2} mv^2$

m = mass of the car in kg
v = velocity of the car at the bottom of the ramp in m/s

The efficiency of the ramp can then be calculated as the ratio of kinetic to potential energy.

Efficiency = KE/PE x 100 to report as a percent

The theoretical maximum velocity can be predicted as well by setting the kinetic energy equal to the potential energy and solving for the velocity.

PE = KE then solving for v gives: $v = (2gh)^{1/2}$

The students can derive this equation easily to show that, in theory, the velocity of the car should depend only on the height of the ramp. It would be simple to stop here and have the students calculate theoretical velocities and actual velocities to calculate the efficiency of the ramp. The ramp however, is a machine designed to translate a vertical force, gravity, into horizontal motion. The students are then told that the ramp, as any other machine, has an optimal set of operating conditions. The conditions are influenced by its surfaces, the operation of the objects it accelerates, in this case little cars, and even the temperature and humidity. It could be presumed that the higher the ramp the faster the car would go at the bottom, and in theory, the maximum speed would be a free fall straight down from a meter high with the ramp at 90 degrees. This angle however, would be outside the useful operating conditions for the ramp, because the little car would just hit the floor and stop. The energy transfer from potential to kinetic energy would only depend on gravity, and not be making use of the ramp as machine to accelerate the car in a horizontal direction. In the same way, the theoretical minimum angle for the ramp would be zero degrees, but in this case there would be no potential energy transfer. This now suggests that for this ramp there is an ideal operating angle, a “sweet spot”, not too steep or not too shallow, where the ramp can translate the most potential energy into kinetic energy for the little car, and the efficiency of the ramp is optimized. The students can measure the angle of the ramp with a protractor, or calculate the angle as the acrtan of the height divided by the depth, assuming the ramp is the hypotenuse. This type of exercise has many practical applications, for example, in landscaping, or in the grading of hills on highways for safety. Also when engineers design any kind of machine, they design for ideal operating conditions, and prepare for actual operation by including a certain amount of “overdesign” to widen the range of the optimal operation of the machine.

Lesson 2: Students will be given a list of required supplies for a camping trip, a budget, a collection of coupons and discounts that may be applied to various items, and a choice of different items, for example, plain tent or fancy tent. They will purchase all the required items while staying within budget. They will document their spending strategy to justify their final shopping list. They may work in teams and may use and excel spreadsheet to calculate all the various combinations. The math in this exercise is simple arithmetic, but the planning involves strategy and perhaps, negotiations among team members. The students should optimize on cost, but also on value, offering justifications for the various tradeoffs in their final plan.

SAMPLE QUESTIONS TO ELICIT CLASS DISCUSSION:

Lesson 1:

- List the various sources of energy for common machines and for each one show how the energy is translated to output.
- Explain efficiency in terms of energy transfer
- What is meant by optimal operating conditions for a machine?

Lesson 2:

- Explain when a coupon for 15% off is better or worse than one for $15 off.
- Explain prioritization, for example, in a shopping list.

HOMEWORK ACTIVITY/EXERCISES/PROBLEMS:

Lesson 1 – Students document the procedures they create to document potential and kinetic energy as a tool to be kept in their reference notebooks. They use it to determine the efficiency of a ramp, including analysis of the velocity of the cars.

Lesson 2 – Student create a shopping list with prices that keep to a budget. The list includes notes on the decisions made and rationale for each item.

PARAMETERS TO EVALUATE STUDENT WORK PRODUCTS:

Lesson 1 – Students do an analysis of the ramp that includes the equations used and derived, rationale for derived equations, results from testing the ramp, and analysis of the results, including an understanding of the boundaries, for example, how the ramp is not 100% efficient.

Lesson 2 – Students prepare a written shopping list includes all required items, is within budget, and includes a rationale for the selection of each item on the list.

This material is based upon work supported by the National Science Foundation under Grant Nos. EEC-0908889

Supporting Program: Center for Pre-College Programs, at the New Jersey Institute of Technology

Contributors

Marie Aloia, Bayonne High School, Bayonne, NJ, Primary Author

Howard Kimmel, Levelle Burr-Alexander, John Carpinelli - Center for pre-College Programs, NJIT.

Dr. C. Ravikumar, Dr. Rajesh Dave - C-SOPS, NJIT

Determining the Optimal Efficiency of a Ramp

Team members ____

Background: When engineers design a machine or an object, or a building for any purpose they design it for an ideal set of operating conditions, with a range of "overdesign". For example, a bridge may be designed for a range of activity, including a typical amount of traffic, a maximum amount of traffic, and maybe, up to a category 4 hurricane. The Verrazano Narrows Bridge was designed for heavy vehicle traffic on 2 decks, but must be monitored carefully when 50-60,000 feet cross it at the start of the NYC marathon, because this kind of traffic is outside of its intended design.

In this exercise you will determine the optimal operation of a ramp, a machine that translates the force of gravity on a toy car into a horizontal velocity. In theory the maximum velocity possible would come from maximum kinetic energy, which in theory is equal to the potential energy of the car at the top of the ramp. Because no translation of energy from one form to another is perfect, according to the second law of thermodynamics, no machine can be 100% efficient. The ramp is not 100% efficient, but, it does, as all machines, have an optimal operating condition based on its design.

In this exercise you will reverse engineer this ramp to find the optimal design conditions where it can translate the most potential energy into kinetic energy.

The formula for potential energy is PE = mgh, where
m equals the mass of the car in kg
g equals the acceleration of gravity, $9.8 m/s^2$, and
h equals the height of the ramp in meters.

The formula for kinetic energy is KE = $\frac{1}{2} mv^2$ where
m equals the mass of the car in kg, and
v equals the velocity of the car in m/s

The efficiency of the ramp could then be expressed as KE/PE x 100.

In theory, at 100% efficiency all the potential energy becomes kinetic energy so the maximum velocity depends only on the height of the ramp.

$$mgh = \frac{1}{2} mv^2 \quad \text{so} \quad v = (2gh)^{1/2}$$

In theory therefore the maximum angle ramp would be 90 degrees, and the minimum angle of the ramp would be zero but neither of these angles would be practical considering that the purpose of the ramp is translate the vertical potential energy, from the force of gravity, into a horizontal velocity. The efficiency can then be expressed as:

Measured velocity / theoretical velocity

At some angle between 0 and 90 degrees there is an optimal energy transfer for this ramp.
Procedure:

1) The ramp is 1 meter long. Mark points on the height of the ramp stand where the angle with the floor is 15 degrees, 30 degrees, 45 degrees, 60 degrees, and 75 degrees.
2) Set a two meter path on the floor in front of the ramp so the car speed can be timed for a fixed distance.
3) Place a paper cover on the ramp and floor path to minimize friction and smooth the transition from the bottom of the ramp to the floor.
4) For each ramp height, measure the time that the car runs down the two meter path at the bottom of the ramp. Run 3 trials, and use the average velocity
5) For each angle and height, calculate the theoretical velocity for each height, then calculate the ramp's efficiency at that height.
6) From the calculated efficiencies at each height speculate the height where the optimal condition might occur, set the amp to that height and re-test the velocity to verify it.
7) Complete the data table and answer the questions

Data Table:

Angle degrees	Theory velocity	Time 1	Time 2	Time 3	Ave Time	Distance 2 m	Ave velocity	efficiency
15								
30								
45								
60								
75								
Test 1								
Test 2								

Test 1 and 2 are for retesting the height where the efficiency appears optimal.

Discussion questions

1) What is the optimal operating angle of this ramp?
2) What features of this ramp and car determine optimal operation?
3) How would the optimal angle change if the ramp was longer? Or shorter? And could you test it with what you have?
4) How would a larger car change the operation?
5) What other factors could change the operation of this ramp and why?

Cost Engineering – Optimizing the Cost and Value for a Camping Trip

Team Members ____

Background: Cost Engineering is a special operation for finding the best value and optimal use of funding, which is one of the most common constraints in any project. In this exercise you will go on an imaginary shopping spree to purchase supplies for a camping trip. You have several bargains available in the form of discounts, or coupons for a fixed amount that may be deducted. You have required and optional items to choose from. You goal is to get all the required items, and any optional items you can afford, while getting the most for your money and staying within your budget.

Procedure:
For this exercise you have a budget. You may spend less than the total, but not more. You must buy all the required items, and you have choices for some different models. You also have a collection of discounts you may apply to any of the items, but only one discount may be applied per item. You may also purchase optional items. Enter each of the items you purchase in the data table. Apply any the discounts to their prices and sum the total. It must be within your budget. Explain your strategy for choices of models and discounts.

Your total budget for this camping trip is: $500.00

You are required to buy the following:

- 1 tent for 4 persons
- mosquito net for the tent
- 4 sleeping bags
- 1 cook stove
- utensils and camp dinnerware for 4
- camping tool kit
- 2 lanterns
- groceries for 1 week – includes non food items such as soap

Optional items: extra lantern, foam mats for sleeping bags, ice cream night at visitors center, binoculars

Discounts: 2 coupons for 20% off, 3 coupons for 10% off, 3 coupons for $20 off

Prices and item models:

- Tents: 1) basic model: $50, 2) model with built in mosquito net $65
  3) deluxe: with mosquito net and padded floor for sleeping $100
- Mosquito net: $20
- Sleeping bag: Basic, to 30F $25, Deluxe arctic down below zero bags $60
- Cook stove: $30
- Camp cooking/eating kit for 4: $40
- Camp tool kit: $60
- Lantern with car charger $20
- Groceries: package includes 3 campsite ready meals/day for 4 people for 1 week: $200
- Foam sleeping mat: $10
- Binoculars: $20
- Ice Cream sundae night at the visitor center $5/person

Research Experiences for Teachers (RET) - 2013

Shopping list plan: (may be done in excel)

Items/ models	Price/model	Discount applied	Final cost
Tent			
Mosquito net			
Sleeping bags			
Cook stove			
Cooking kit			
Tool kit			
Lanterns			
Groceries			

Total final cost _______________

Cost Strategy Questions

1) How close does your estimate come to the required budget?
2) Explain the reasons behind each of your choices of models optional items and how it adds value to your estimate.
3) Which discount/saving option had the most impact on your estimate?