NJIT New Jersey Institute of Technology
Center for Pre-College Programs
Campbell Hall, 4th & 5th Floors
University Heights
Newark, New Jersey 07102-1982
Phone: 973-596-3550
njit.edu/precollege

Helping Kids Learn – Post #11 4/28/20

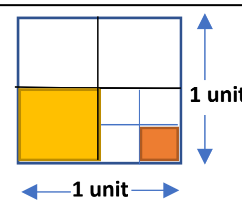
STEM: Mathematics – Operations with Fractions

If you cook, chances are you don't need a recipe for foods you prepare frequently. Even then, though, you may not remember if you add 2 teaspoons of baking powder and 1 teaspoon of baking soda or the other way around. If you stop to think about the taste or you know something about baking chemistry, you may know the answer without looking at the recipe. That's because you can attach meaning to the ingredients beyond their quantity. The same is true of multiplying and dividing fractions as you'll see in these investigations. To add even more context, see Lift the Level below.

Be a Mathematician – Making sense of multiplying and dividing fractions

When you learn to multiply, it is almost always with whole numbers. So, except when either 0 or 1 is a factor, the product (answer) is a greater amount than either factor: **6 X 4 = 24** 24 is greater than either 6 or 4.

By the time you learn to multiply fractions, you know multiplication so well that it seems wrong that the product would be *a smaller amount* than either factor. But that's just what happens when the factors are two fractions less than 1.



Picture 1

Investigation 1: Find the area of the yellow part of the unit square in Picture 1.

First, look at it. What is a reasonable answer? ____________unit You can see that it fills about ¼ of the square.

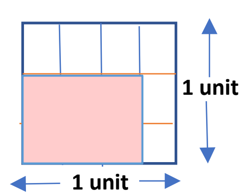
The area of a square or a rectangle is its length times its width: **A = ℓ x w**

The black lines split the square into 4 equal pieces. The sides of the yellow part (a square, too) are ½ unit long by ½ unit wide. So, the area of the yellow part is **½ x ½ = ¼ of the whole square.**

Investigation 2: What is the area of the orange part of the unit square?

Again, first think what is reasonable. ___________________unit You might think, "It's smaller than the yellow part." Or "It looks about ¼ as big as the yellow part." Or "Each side looks about ¼ as long as the whole square." Or "It would take 16 orange squares to fill the whole square. So, orange is $^{1}/_{16}$ of the whole square."

The sides of the orange square are ¼ unit long by ¼ unit wide. So, the area of the orange part is **¼ x ¼ = $^{1}/_{16}$ of the whole square.**

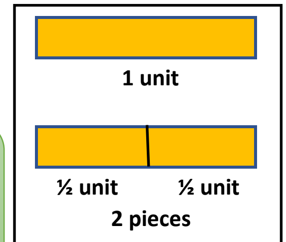


Picture 2

Investigation 3: Find the area of the pink part of the unit square in Picture 2. The blue lines split the unit square into **fourths**, the orange lines split it into **thirds.** The pink part is a rectangle $^{3}/_{4}$ unit by $^{2}/_{3}$ unit.

(See Solutions below.)

Investigation 4: Division, or dividing, is a way of splitting things up. Imagine we have a gold bar that we want to divide in half. (See Post #5 in this series.) How many halves of gold bar would we get? Picture 3 shows that $\frac{1}{\frac{1}{2}}$ **= 2.**



Picture 3

1 divided by ½ is 2. There are two ½s in 1.

Investigation 5: Find $\frac{3/4}{1/2}$.

In other words, how many ½s are in ¾?
There is one ½ unit and half of another or 1½. See Picture 4.

Picture 4

¾ divided by ½ is 1½.

Lift the Level
You can make this lesson deeper or suitable for younger students by the following:

1. We used an area model (squares) for multiplication of fractions and a linear model (bars) for division of fractions. But you could reverse the two. Try it for yourself.
2. We didn't jump from a few examples to a general *algorithm* (a step-by-step method for solving a problem). Use the examples in the Investigations to write an ***algorithm for multiplying two fractions*** and an ***algorithm for dividing two fractions.*** An algorithm should work for all kinds of fractions: less than 1, greater than 1, positive, and negative.
3. In Lift the Level #2, use your division algorithm to show why we say you cannot divide by zero.
4. Write a word problem that uses ¾ x ⅔ to solve it. Write other word problems that use multiplication of fractions to solve them. Give them to a friend or family member to solve. [This is the best way to learn if you wrote something that doesn't make sense to someone else!]
5. Complete Lift the Level #4 for ¾ divided by ½ and other division of fractions.
6. For younger students: Use the fraction bars from Post #5 in this series. Fold the bars or cut one set and put pieces on top of one another. Do lots of division problems this way before formalizing with an algorithm (see Lift the Level #2 above).
7. Investigations 1 and 2 use *square roots* and *square numbers.* Just as the names suggests, a *square root* is the basis for a square – the length of the side (all sides of a square are the same length). And a *square number* is the area of a square whose side is the square root long. Make a table of fractional square roots and squares and illustrating each with an appropriately labeled square.

NJ Student Learning Standards **Mathematics**

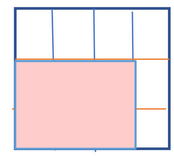
There's a set of "umbrella" mathematics standards that are the foundation for the other standards. For this investigation, the following Standards for Mathematical Practice are addressed:

In addition to the umbrella standards, the following Numbers standards are addressed:

3.NF.A.3, 4.NF.B, 5.NF.B.4, 5.NF.B.5, 6.NS.A.1, 7.NS.A.2

Solutions

Investigation 3: The pink part is a rectangle $^{3}/_{4}$ unit by $^{2}/_{3}$ unit.



¾ x ⅔ = $^{6}/_{12}$ or ½. You can count to verify: 6 of the 12 pieces are pink (and 6 white).